In this homework, the energy conservation and energy concentration properties of the following three transforms are compared:

- Haar Transform (HT)
- Discrete-Cosine Transform (DCT)
- Karhunen-Loeve Transform (KLT)

Use ‘mri.tif’, ‘einstein.tif’ and ‘smandril.tif’ as the test image set. Note that in the following questions we work on the whole image set, not on the individual image.

1. Carry out HT and DCT for each 8x8 block in the image set. Measure the mean squared value of each transform coefficient by averaging over all blocks in the entire data set. (you get 64 numbers for HT and 64 numbers for DCT).

   (a) Measure and compare the average energy of the image set and that of the transform coefficients. (compare 3 numbers here, from the image set, the HT coefficients, and the DCT coefficients). For HT and DCT, do they have energy conservation property?

   (b) Order the transform coefficients by descending mean squared values. For each of HT and DCT, generate an 8x8 matrix with elements indicating the order of the transform coefficient mean squared values. Specifically, with ‘1’ at the position of the coefficient which has the largest mean squared value; and ‘64’ at the position of the coefficient which has the smallest.

   (c) Generate a cumulative energy plot that shows the percentage of the total energy contained in the subset of the first \( k \) coefficient as a function of \( k \).

2. Measure the autocorrelation matrix of the pixels within an 8x8 block by averaging over all blocks in the image set.

   (a) How can you calculate the percentage of the total energy that is contained in the subset of the first \( k \) coefficients of an 8x8 KLT directly from the autocorrelation matrix without actually carrying out a KLT?

   (b) Does KLT have energy conservation property? Also compare the energy concentration of KLT with that of HT and DCT by showing the three curves of cumulative energy in the same graph.

Note: Some MATLAB functions listed below might be useful for this homework.

dct2( ), sort( ), flipud( ), reshape( ), cumsum( ), eig( )